EXPERIMENTS IN SQUARE LATTICE WITH A COMMON TREATMENT IN ALL BLOCKS

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INTRODUCTION

Since a long time ago, the **Instituto Agronômico de Campinas**, a research institute located em Campinas, São Paulo, Brasil, uses for its experiments with corn (maize) varieties and hydrids, square lattices with k^2 treatments and blocks of $k + 1$ plots, the extra plot in each block receiving a standard variety or hybrid, the same for all blocks, not included among the k^2 original treatments. It is clear, therefore, that these square lattice experiments include, on the whole, $k^2 + 1$ treatments in blocks of $k + 1$ plots. For example, in a 3^2 lattice, with 2 orthogonal replications, and treatments 1, 2, ..., 9, plus treatment A (standard variety), the blocks would be as follows:

Block 1 : 1 2 3 A
Block 2 : 4 5 6 A
Block 3 : 7 8 9 A
Block 4 : 1 4 7 A
Block 5 : 2 5 8 A
Block 6 : 3 6 9 A

This paper deals with the intrablock analysis of these designs.

INTRABLOCK ANALYSIS

In these designs we have $v = k^2 + 1$ treatments in a square lattice with **m** orthogonal replications, $b = mk$ blocks, of $k + 1$ plots.

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There are k^2 regular treatments (1, 2, ..., k^2), plus a common treatment A. The parameter λ_{ij} is equal to 1 for regular treatments that appear in the same block (first associates), equal to zero for regular treatments that do not appear in the same block (second associates). But when one of the treatments is the common one, then $\lambda_{ij} = m$. So, the normal equations (KEMPTHORNE, 1952; PIMENTEL GOMES, 1969) will have coefficients.

$$c_{ii} = m\left(1 - \frac{1}{k+1}\right) = \frac{mk}{k+1},$$

$$c_{ij} = -\frac{1}{k+1} \quad \text{(first associates)},$$

$$c_{ij} = 0 \quad \text{(second associates)},$$

$$c_{iA} = -\frac{m}{k+1} \quad \text{(a regular treatment and the common treatment)},$$

$$c_{AA} = mk\left(1 - \frac{1}{k+1}\right) = \frac{mk^2}{k+1}.$$

For the example above, the equation corresponding to treatment 1 is:

$$\frac{mk}{k+1}t_1 - \frac{1}{k+1}t_2 - \frac{1}{k+1}t_3 - \frac{1}{k+1}t_4 - \frac{1}{k+1}t_7 - \frac{m}{k+1}t_A = Q_1$$

and for treatment A (common) it is:

$$-\frac{m}{k+1}t_1 - \frac{m}{k+1}t_2 - \dots - \frac{m}{k+1}t_9 + \frac{mk^2}{k+1}t_A = Q_A.$$

Since matrix $C = (c_{ij})$ is singular, we introduce a restriction for the treatment effects, which can be

$$\sum_{i=1}^{k^2} t_i + kt_A = 0.$$

The solution of the system of normal equation gives then:

$$t_A = \frac{1}{mk} Q_A ,$$

$$t_i = \frac{1}{m} Q_i + \frac{1}{k(mk - k + m)} Q_A +$$

$$+ \frac{1}{m(mk - k + 1)} \left[S_1(Q_i) + S_2(Q_i) + \ldots + S_m(Q_i) \right] ,$$

where $S_j(Q_i)$ is the sum of Q's in the j^{th} replication, in the block where the i^{th} treatment appears.

The sum of squares for treatments (adjusted) is:

$$SST(adjusted) = \frac{1}{m} \Sigma Q_i^2 + \frac{m-1}{m(mk - k + 1)} Q_A^2 +$$

$$+ \frac{1}{m(mk - k + m)} \sum_{j,j'} S_{jj'}^2(Q) ,$$

where $S_{jj'}(Q)$ is the sum of Q's in the j'th block of the j^{th} replication. So the analysis of variance is obtained as explained in table I.

TABLE I - Analysis of variance.

Source of variation	D.F.	S.S.
Replications	m - 1	As usual
Blocks within replications (unadjusted)	m (k - 1)	As usual
Treatments (adjusted)	k^2	By formula
Residual	By subtraction	By subtraction
Total	m k (k + 1) - 1	As usual

The adjusted treatment means are:

$$\hat{m}_i = \frac{G}{mk\,(k+1)} + \hat{t}_1 \quad , \hat{m}_A = \frac{G}{mk\,(k+1)} + \hat{t}_A \ ,$$

where G is the grand total of all plots.

CONTRASTS BETWEEN TREATMENT MEANS

There are 3 cases to be studied.

1st associates: Two regular treatments ocurring in the same block, for instance, treatments 1 and 2 in the example above:

$$V(\hat{m}_i - \hat{m}_j) = \frac{2\,\sigma^2}{m}\left[1 + \frac{m-1}{mk-k+m}\right].$$

2st associates: Two regular treatments which do not occur in the same block, for instance treatments 1 and 6 in the example above:

$$V(\hat{m}_i - m_j) = \frac{2\,\sigma^2}{m}\left[1 + \frac{m}{mk-k+m}\right].$$

3rd associates: A regular treatment and the common treatment:

$$V(\hat{m}_i - \hat{m}_A) = \sigma^2\left[\frac{1}{m} + \frac{1}{mk} + \frac{k-1}{k(mk-k+m)}\right].$$

EXAMPLE OF ANALYSIS

We shall take as example a 5 x 5 aquare lattice with 4 orthogonal replications and a common treatment A present in all blocks. The experiment was carried out with corn (maize), and harvest expressed in kg./ha. (table II).

TABLE II - Yields, in kg/ha, of corn (maize) in the trial in the quadruple square lattice, with a common treatment in all blocks, used as example.

Block Number							Block Totals
1st replicate							
1	6126 (5)	6497 (11)	6309 (8)	6271 (19)	5743 (22)	6602 (A)	37548
2	6809 (2)	6642 (10)	5111 (13)	4646 (24)	5240 (16)	6173 (A)	34621
3	3670 (4)	6899 (7)	5770 (21)	4167 (18)	4195 (15)	6430 (A)	31131
4	6610 (12)	7166 (20)	5925 (23)	5332 (1)	5509 (9)	4608 (A)	35150
5	6175 (14)	7413 (3)	5768 (6)	6059 (17)	5704 (25)	7202 (A)	38321
							176771
2nd replicate							
6	6218 (1)	6692 (18)	6621 (10)	6321 (14)	6318 (22)	5787 (A)	37957
7	5580 (2)	5586 (15)	4682 (23)	6155 (19)	8237 (6)	5487 (A)	35727
8	6199 (25)	3844 (4)	5549 (12)	5550 (16)	6480 (8)	4844 (A)	32466
9	5960 (11)	5019 (24)	6300 (7)	6993 (20)	5996 (3)	6280 (A)	36548
10	6986 (17)	5191 (9)	7204 (5)	6999 (21)	6394 (13)	6001 (A)	38775
							181473
3rd replicate							
11	6350 (25)	6519 (1)	5195 (13)	6187 (19)	5412 (7)	4415 (A)	34078
12	4542 (8)	4330 (8)	5580 (2)	3847 (21)	4339 (14)	5273 (A)	27911
13	4491 (4)	6285 (17)	4927 (11)	3998 (23)	3846 (10)	6044 (A)	29591
14	5374 (22)	5690 (15)	4230 (3)	4177 (9)	5416 (16)	5165 (A)	30052
15	6064 (6)	5992 (18)	5780 (5)	5102 (24)	4692 (12)	4887 (A)	31817
							153449
4th replicate							
16	6052 (7)	6439 (6)	6600 (10)	5855 (8)	7160 (9)	6687 (A)	38793
17	4125 (1)	5822 (2)	2956 (4)	7022 (5)	7804 (3)	5273 (A)	33002
18	4235 (13)	4867 (11)	4734 (15)	6342 (14)	7691 (12)	5744 (A)	33613
19	5199 (16)	3985 (20)	5029 (19)	4998 (18)	6223 (17)	4823 (A)	30257
20	5129 (29)	5880 (25)	3609 (24)	5718 (23)	5538 (22)	6044 (A)	31118
							166783

For treatments 1, 2, ...,25 and A we compute now the totals T_i (i= 1, 2, ..., 25, A) and the adjusted treatment totals.

$$Q_i = T_i - \sum_j{}' \frac{n_{ij}}{k+1} B_j ,$$

TABLE III - Data analyzed, with values of T_i, Q_i and $\hat{m}_i$ for each treatment.

Treat. n.º	1st rep.	2nd rep.	3rd rep.	4th rep.	T_i	Q'_i	$\hat{m}_i$
1	5332	6218	6519	4125	22,194	- 7023	5318
2	6809	5580	5580	5822	23,791	11485	6146
3	7413	5996	4230	7804	25,443	14735	6369
4	3670	3844	4491	2956	14,961	-36424	3948
5	6126	7204	5780	7022	26,132	15650	6406
6	5768	8237	6064	6439	26,508	14390	6466
7	6899	6300	5412	6052	24,663	7428	5899
8	6309	6480	4542	5855	23,186	2398	5846
9	5509	5191	4177	7160	22,037	-10548	5241
10	6642	6621	3846	6600	23,709	1292	5606
11	6497	5960	4927	4867	22,251	- 3794	5440
12	6610	5549	4692	7691	24,542	14206	6211
13	5111	6394	5195	4235	20,935	-15477	4927
14	6175	6321	4339	6342	23,177	1260	5829
15	4195	5586	5690	4734	20,205	- 9293	5175
16	5240	5550	5416	5199	21,405	1034	5660
17	6059	6986	6285	6223	25,553	16374	6451
18	4167	6692	5892	4998	21,749	- 668	5636
19	6271	6155	6187	5029	23,642	4242	5941
20	7166	6993	4330	3985	22,474	4978	5948
21	5770	6999	3847	5129	21,745	1535	5642
22	5743	6318	5374	5538	22,973	1163	5673
23	5925	4682	3998	5718	20,323	- 9648	5130
24	4646	5019	5102	3609	18,376	-23848	4589
25	5704	6199	6350	5080	23,333	4015	5827
A	6602	5787	4415	6687			
	6173	5487	5273	5273			
	6430	4844	6044	5744			
	4608	6280	5165	4823			
	7202	6001	4287	6044	113,169	538	5658
	176771	181473	153449	166783			

where N= (n_{ij}) is the **incidence matrix**, and B_j is the total of block **j**. It is known that the incidence matrix is obtained from elements n_{ij}, with n_{ij}= 1 if treatment **i** occurs in block **j**, and n_{ij}= 0, if it does not occur.

However, it is easier to calculate $Q_i' = (k + 1) Q_i$, as done in table III. We have:

$$T_1 = 22194$$

$$Q_1' = 6T_1 - (B_4 + B_6 + B_{11} + B_{17})$$

$$= 6 \times 22194 - (35150 + 37957 + 34078 + 33002)$$

$$= -7023$$

$$Q_A' = 6\,Q_A = 6 \times 113169 - 678476$$

$$= 538.$$

Formulas given in secction 2 can be easily changed to use Q'– values instead of Q - values. We obtain:

$$t_A = \frac{1}{mk(k+1)} Q_A'$$

$$= (1/120)\ 538$$

$$= 4.5 \approx 4 \ .$$

$$t_1 = \frac{1}{m(k+1)} Q_j' + \frac{1}{k(k+1)(mk-k+m)} Q_A' +$$

$$+ \frac{1}{(k+1)\,m(mk-k+m)} \left[S_1(Q_1') + S_2(Q_1') + \ldots + S_m(Q_1') \right]$$

$$= (1/24)(-7023) + (1/570)(538) + (1/456)(-20403)$$

$$= -336.$$

On the other hand, we have

$$\hat{m}_1 = 5654 - 336 = 5318,$$

$$\hat{m}_A = 5654 + 4 = 5658.$$

The sum of squares for treatments (adjusted) is:

$$SST(\text{adjusted}) = \frac{1}{m\,(k+1)^2}\,\Sigma\, Q_i'^2 + \frac{m-1}{m(k+1)^2\,(mk-k+m)}\,Q_A'^2 +$$

$$+ \frac{1}{m(k+1)^2\,(mk-k+m)} \sum_{j,j'} S^2_{jj'}\,(Q_i')$$

$$\acute{S}ST(\text{adjusted}) = (1/144)\ 3{,}888{,}633{,}692 + (1/912)\ 289{,}444 +$$

$$+ (1/2736)\ 9{,}235{,}411{,}576 = 30{,}380{,}234.$$

The analysis of variance obtained is given in table IV.

TABLE IV - Analysis of variance of data in table II.

Source of variation	D.F.	S.S.	M.S.	F
Blocks	19	35,779,509		
Treatments (adjusted)	25	30,380,234	1,215,209	1,67*
Error	75	54,631,221	728,416	

For first associates we have:

$$\hat{V}(\hat{m}_i - \hat{m}_j) = \frac{2(728416)}{4} \quad (1 + 3/19)$$

$$= (0,5789)\ 728416$$
$$= 421680.$$

For second associates the estimate of variance is:

$$\hat{V}(\hat{m}_i - \hat{m}_u) = \frac{2(728416)}{4} \quad (1 + 4/19)$$

$$= (0,6053)\ 728416$$
$$= 440910.$$

Finally, for a contrast between the common treatment and any other treatment we have:

$$\hat{V}(\hat{m}_i - \hat{m}_A) = 728416\ \left(\frac{1}{4} + \frac{1}{20} + \frac{1}{76} \right)$$

$$= 247278.$$

For the usual 5 x 5 aquare lattice we should obtain

$$\hat{V}(\hat{m}_i - \hat{m}_u) = 0,6000\ s^2$$

for first associates, and

$$\hat{V}(\hat{m}_i - \hat{m}_u) = 0,6333\ s^2$$

for second associates. We conclude, therefore, that the new design gives lower estimates for these variances.

SUMMARY

This paper deals with a generalization of square lattice designs, with k^2 treatments in blocks of k + 1 plots, the extra plot in each block receiving a standard treatment, the same for all blocks. The new design leads to lower variances for contrasts between adjusted treatment means.

RESUMO

O Instituto Agronômico de Campinas vem, há muitos anos, utilizando nos seus ensaios de milho, reticulados quadrados com k^2 tratamentos em blocos de k + 1 parcelas, sendo a parcela extra de cada bloco cultivada com um cultivar padrão (variedade ou híbrido), não incluído entre os k^2 tratamentos originais. Conclui-se, pois, que esses delineamentos incluem k^2 + 1 tratamentos, em blocos de k + 1 parcelas.

O presente trabalho deduz fórmulas para a análise da variância desses delineamentos, e para a estimação das médias ajustadas de tratamentos. Fórmulas para a variância de diversos contrastes são deduzidas. Finalmente, apresenta-se um exemplo, detalhadamente analisado, de um ensaio em reticulado quadrado com k^2= 25, e 4 repetições ortogonais, instalado com 26 cultivares, em blocos de 6 parcelas.

LITERATURE CITED

KEMPTHORNE, O., 1952. **The design and analysis of experiments**, John Wiley Sons, New York.

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